

JEE Maths: Trigonometry & Calculus Sheet

Class 11 & 12 • JEE Main & Advanced | by Anish Kumar Sir

Trigonometric Identities

TOPIC	FORMULA	CONCEPT
Compound angles	$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	Expands the sine of a sum or difference.
	$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	Note the sign flips for cosine.
$\tan(A \pm B)$	$(\tan A \pm \tan B) / (1 \mp \tan A \tan B)$	$\tan(45^\circ + A) = (1 + \tan A)/(1 - \tan A)$.
Double angle	$\sin 2A = 2\sin A \cos A = 2\tan A/(1 + \tan^2 A)$	Converts products into a single angle.
	$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$	Gives $\sin^2 A = (1 - \cos 2A)/2$, $\cos^2 A = (1 + \cos 2A)/2$.
	$\tan 2A = 2\tan A / (1 - \tan^2 A)$	Undefined when $\tan A = \pm 1$.
Triple angle	$\sin 3A = 3\sin A - 4\sin^3 A$; $\cos 3A = 4\cos^3 A - 3\cos A$	$\tan 3A = (3\tan A - \tan^3 A)/(1 - 3\tan^2 A)$.
Sum to product	$\sin C + \sin D = 2\sin((C+D)/2) \cos((C-D)/2)$	Turns sums into products for factorising.
	$\cos C + \cos D = 2\cos((C+D)/2) \cos((C-D)/2)$	$\cos C - \cos D = -2\sin((C+D)/2) \sin((C-D)/2)$.
Product to sum	$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$2\cos A \cos B = \cos(A+B) + \cos(A-B)$.
Range	$-\sqrt{a^2 + b^2} \leq a \sin \theta + b \cos \theta \leq \sqrt{a^2 + b^2}$	Frequently asked for max/min questions.
Useful products	$\sin \theta \sin(60^\circ - \theta) \sin(60^\circ + \theta) = \frac{1}{4} \sin 3\theta$	Same pattern holds for cos and tan.

Trigonometric Equations & Triangles

TOPIC	FORMULA	CONCEPT
$\sin \theta = \sin \alpha$	$\theta = n\pi + (-1)^n \alpha$	$n \in \mathbb{Z}$; general solution covers all periods.
$\cos \theta = \cos \alpha$	$\theta = 2n\pi \pm \alpha$	Cosine is even, hence $\pm \alpha$.
$\tan \theta = \tan \alpha$	$\theta = n\pi + \alpha$	$\tan$ has period π .
$\sin^2 \theta = \sin^2 \alpha$	$\theta = n\pi \pm \alpha$	Same holds for $\cos^2$ and $\tan^2$.
Sine rule	$a/\sin A = b/\sin B = c/\sin C = 2R$	R is the circumradius.
Cosine rule	$\cos A = (b^2 + c^2 - a^2) / 2bc$	Finds an angle from three sides.
Area of triangle	$\Delta = \frac{1}{2}bc \sin A = \sqrt{s(s-a)(s-b)(s-c)}$	$s = (a + b + c)/2$ (Heron's formula).
Projection formula	$a = b \cos C + c \cos B$	Side equals sum of projections of the other two.
Inverse trig	$\sin^{-1} x + \cos^{-1} x = \pi/2$; $\tan^{-1} x + \tan^{-1} y = \tan^{-1}[(x+y)/(1-xy)]$ ($xy < 1$)	Principal ranges: $\sin^{-1} [-\pi/2, \pi/2]$, $\cos^{-1} [0, \pi]$.

Limits & Continuity

TOPIC	FORMULA	CONCEPT
Standard limits	$\lim_{x \rightarrow 0} \sin x/x = 1$, $\lim_{x \rightarrow 0} \tan x/x = 1$ ($x \rightarrow 0$)	x must be in radians.

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TOPIC	FORMULA	CONCEPT
Exponential / log	$\lim (e^x - 1)/x = 1$, $\lim (a^x - 1)/x = \ln a$, $\lim \ln(1+x)/x = 1$	Use when expression is $0/0$ near $x = 0$.
1^∞ form	$\lim f^g = e^{(\lim g(f-1))}$	Shortcut for limits like $(1 + 1/x)^x \rightarrow e$.
Power form	$\lim (x^n - a^n)/(x - a) = na^{n-1} (x \rightarrow a)$	Behind the derivative of x^n .
L'Hôpital's rule	$\lim f/g = \lim f'/g'$ for $0/0$ or ∞/∞	Differentiate numerator and denominator separately.
Series expansions	$e^x = 1 + x + x^2/2! + \dots$; $\sin x = x - x^3/3! + \dots$; $\cos x = 1 - x^2/2! + \dots$	Fastest tool for tricky limits.
Continuity	$LHL = RHL = f(a)$	Differentiable $\Rightarrow$ continuous, not the reverse (e.g. $ x $ at 0).

Differentiation

TOPIC	FORMULA	CONCEPT
Power & exponential	$d/dx x^n = nx^{n-1}$, $d/dx e^x = e^x$, $d/dx a^x = a^x \ln a$	Exponential is its own derivative.
Logarithm	$d/dx \ln x = 1/x$, $d/dx \log_a x = 1/(x \ln a)$	Logarithmic differentiation handles x^x type functions.
Trig	$(\sin x)' = \cos x$, $(\cos x)' = -\sin x$, $(\tan x)' = \sec^2 x$	$(\sec x)' = \sec x \tan x$, $(\cot x)' = -\operatorname{cosec}^2 x$.
Inverse trig	$(\sin^{-1} x)' = 1/\sqrt{1-x^2}$, $(\tan^{-1} x)' = 1/(1+x^2)$	$(\cos^{-1} x)'$ and $(\cot^{-1} x)'$ are their negatives.
Product & quotient	$(uv)' = u'v + uv'$; $(u/v)' = (u'v - uv')/v^2$	Quotient rule: "low d-high minus high d-low".
Chain rule	$dy/dx = (dy/du)(du/dx)$	Differentiate outside to inside.
Parametric	$dy/dx = (dy/dt)/(dx/dt)$	Used for projectile and curve problems.
Tangent & normal	Slope $m = dy/dx$; normal slope $= -1/m$	Tangent: $y - y_1 = m(x - x_1)$.
Maxima / minima	$f'(x) = 0$; $f'' < 0$ max, $f'' > 0$ min	Also check end points on a closed interval.
Rate of change	$dA/dt = (dA/dr)(dr/dt)$	Physics link: $v = dx/dt$, $a = dv/dt$.

Integration

TOPIC	FORMULA	CONCEPT
Power rule	$\int x^n dx = x^{n+1}/(n+1) + C$ ($n \neq -1$); $\int dx/x = \ln x + C$	Reverse of differentiation.
Exponential	$\int e^x dx = e^x + C$, $\int a^x dx = a^x/\ln a + C$	$\int e^x [f(x) + f'(x)] dx = e^x f(x) + C$.
Trig	$\int \sin x = -\cos x$, $\int \cos x = \sin x$, $\int \sec^2 x = \tan x$	$\int \tan x = \ln \sec x $, $\int \sec x = \ln \sec x + \tan x $.
Standard forms	$\int dx/(x^2 + a^2) = (1/a)\tan^{-1}(x/a) + C$	$\int dx/\sqrt{a^2 - x^2} = \sin^{-1}(x/a) + C$.
	$\int dx/(x^2 - a^2) = (1/2a)\ln (x - a)/(x + a) + C$	$\int dx/\sqrt{x^2 \pm a^2} = \ln x + \sqrt{x^2 \pm a^2} + C$.
By parts	$\int u dv = uv - \int v du$	Choose u by ILATE: Inverse, Log, Algebraic, Trig, Exponential.

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TOPIC	FORMULA	CONCEPT
Substitution	$x = a \sin \theta$ for $\sqrt{(a^2 - x^2)}$; $x = a \tan \theta$ for $a^2 + x^2$	Turns roots into trig identities.
Definite: King's rule	$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$	Solves many symmetric JEE integrals in one line.
Odd / even	$\int_{-a}^a f = 0$ (odd); $= 2 \int_0^a f$ (even)	Check symmetry before integrating.
Periodic	$\int_0^{nT} f(x) dx = n \int_0^T f(x) dx$	T is the period of f.
Area under curve	Area $= \int_a^b f(x) dx$	Between curves: $\int (\text{upper} - \text{lower}) dx$.
Differential equations	$dy/dx + Py = Q$: IF $= e^{\int P dx}$, $y \cdot \text{IF} = \int Q \cdot \text{IF} dx$	Linear first-order form; separable: $\int dy/g(y) = \int f(x) dx$.

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